JEE Advanced 2026

Sample Paper - 4 (Paper-2)

Time Allowed: 3 hours **Maximum Marks: 180**

General Instructions:

This question paper has THREE main sections and three sub-sections as below.

MCQ

- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- You will get +3 marks for the correct response and -1 for the incorrect response.

MRQ

- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) the correct answer(s).
- You will get +4 marks for the correct response and -2 for the incorrect response.
- You will also get 1-3 marks for a partially correct response.

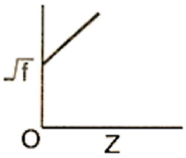
NUM

- The answer to each question is a NON-NEGATIVE INTEGER.
- You will get +4 marks for the correct response and 0 marks for the incorrect response.

Physics

1. Identify the graph which correctly represents the Moseley's law: **[3]**

a)



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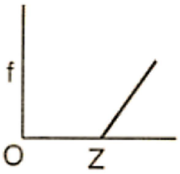

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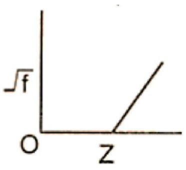

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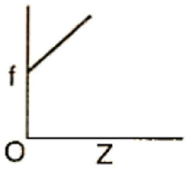
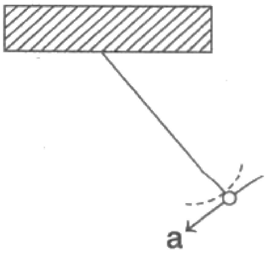


2. A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector $\vec{a}$ is correctly shown in **[3]**

a)



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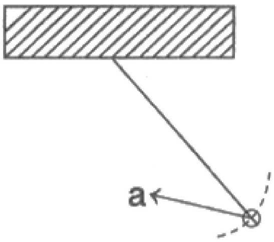

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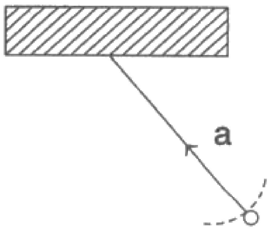

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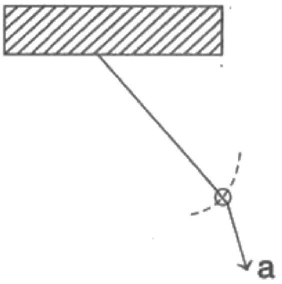


3. The energy of a photon is equal to the kinetic energy of a proton. The energy of the photon is E. Let λ_1 be the de-Broglie wavelength of the proton and λ_2 be the wavelength of the photon. The ratio $\frac{\lambda_1}{\lambda_2}$ is proportional to **[3]**

a) E^{-2}

b) E^{-1}

c) $E^{1/2}$

d) E^{0}

4. During a projectile motion, if the maximum height equals to the horizontal range, then the angle of projection with the horizontal is: **[3]**

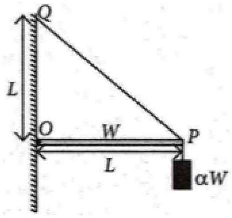
a) $\tan^{-1}(3)$

b) $\tan^{-1}(4)$

c) $\tan^{-1}(1)$

d) $\tan^{-1}(2)$

5. One end of a horizontal uniform beam of weight W and length L is hinged on a vertical wall at point O and its other end is supported by a light inextensible rope. The other end of the rope is fixed at point Q, at a height L above the hinge at point O. A block of weight αW is attached at the point P of the beam, as shown in the figure (not to scale). The rope can sustain a maximum tension of $(2\sqrt{2})W$. **[4]**



Which of the following statement(s) is(are) correct?

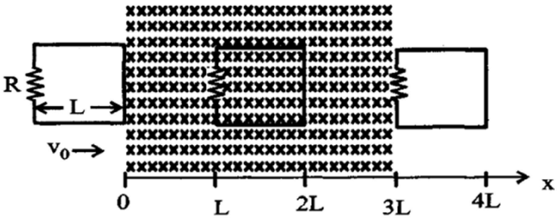
a) The vertical component of reaction force at O does not depend on α.

b) The tension in the rope is 2W for $\alpha = 0.5$

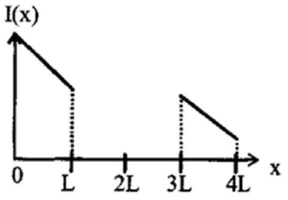
c) The rope breaks if $\alpha > 1.5$

d) The horizontal component of reaction force at O is equal to W for $\alpha = 0.5$

6. A rigid wire loop of square shape having side of length L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper. At t = 0, the right edge of the loop enters a region of length 3L where there is a uniform magnetic field B_0 into the plane of the paper, as shown in the figure. For sufficiently large v_0. the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let v(x), I(x) and F(x) represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of x. Counter-clockwise current is taken as positive. **[4]**



Which of the following schematic plot(s) is (are) correct? (Ignore gravity)

a)


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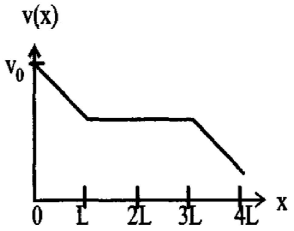
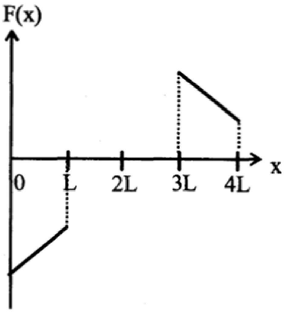


c)


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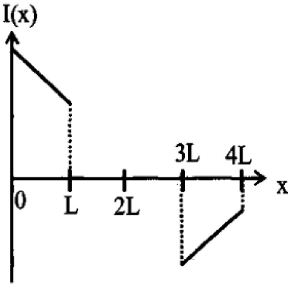
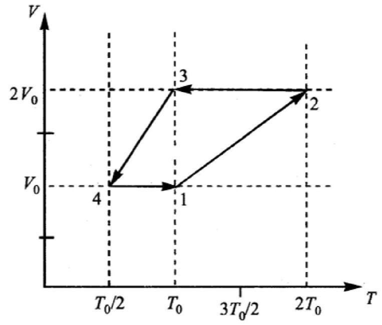


7. One mole of a monatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature (V - T) diagram. The correct statement(s) is/are: **[4]**

[R is the gas constant]



a) Work done in this thermodynamic cycle ($1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1$) is $|W| = \frac{1}{2}RT_0$

b) The ratio of heat transfer during processes $1 \rightarrow 2$ and $3 \rightarrow 4$ is $\left|\frac{Q_{1\rightarrow 2}}{Q_{2\rightarrow 3}}\right| = \frac{1}{2}$

c) The above thermodynamic cycle exhibits only isochoric and adiabatic processes

d) The ratio of heat transfer during processes $1 \rightarrow 2$ and $3 \rightarrow 4$ is $\left|\frac{Q_{1\rightarrow 2}}{Q_{2\rightarrow 3}}\right| = \frac{5}{3}$

8. A simple pendulum of length L and mass (bob) M is oscillating in a plane about a vertical line between angular limit $-\phi$ and $+\phi$. For an angular displacement $\theta(|\theta| < \phi)$, the tension in the string and the velocity of the bob are T and V respectively. The following relations hold good under the above conditions: **[4]**

a) The magnitude of the tangential acceleration of the bob $|a_T| = g \sin\theta$

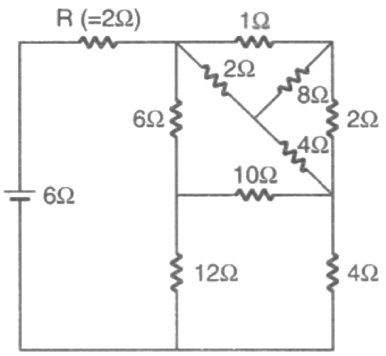
b) 7 - Mgcos$\theta = \frac{MV^2}{L}$

c) $T = Mg\cos\theta$

d) T cosθ = Mg.

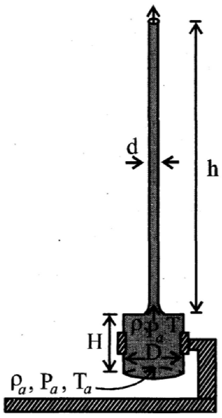
9. The distance between two stars of masses $3M_S$ and $6M_S$ is 9 R. Here R is the mean distance between the centers of the Earth and the Sun, and M_S is the mass of the Sun. The two stars orbit around their common center of mass in circular orbits with period nT, where T is the period of Earth's revolution around the Sun. The value of n is ________. **[4]**

10. In the adjoining circuit, the current through the resistor R(= 2Ω) is I amperes. The value of I is: **[4]**



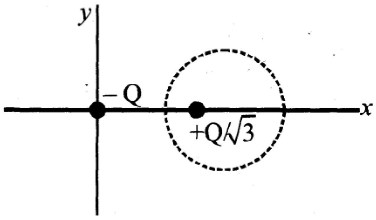
11. A cylindrical furnace has height (H) and diameter (D) both 1 m. It is maintained at temperature 360 K. The air gets heated inside the furnace at constant pressure P_a and its temperature becomes T = 360 K. The hot air with density ρ rises up a vertical chimney of diameter d = 0.1 m and height h = 9 m above the furnace and exits the chimney (see the figure). As a result, atmospheric air of density ρ_a = 1 .2 kg m^{-3}, pressure P_a and temperature T_a = 300 K enters the furnace. Assume air as an ideal gas, neglect the variations in ρ and T inside the chimney and the furnace. Also ignore the viscous effects. **[4]**

[Given: The acceleration due to gravity g = 10 ms^{-2} and π = 3.14]



Considering the air flow to be streamline, the steady mass flow rate of air exiting the chimney is ________ gm s^{-1}.

12. Two point charges -Q and $+\frac{Q}{\sqrt{3}}$ are placed in the xy-plane at the origin (0, 0) and a point (2, 0), respectively, as shown in the figure. This results in an equipotential circle of radius R and potential V = 0 in the xy-plane with its center at (b, 0). All lengths are measured in meters. **[4]**



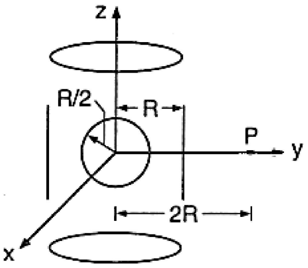
The value of R is ________ meter.

13. Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8 Nm^{-2}. The radii of bubbles A and B are 2 cm, respectively. The surface tension of the soap water used to make bubbles is 0.04 Nm^{-1}. Find the ratio $\frac{n_B}{n_A}$, where n_A and n_B are the number of moles of air in n_A bubbles A and B, respectively. [Neglect the effect of gravity] **[4]**

14. In a radioactive decay process, the activity is defined as $A = -\frac{dN}{dt}$, where N(t) is the number of radioactive nuclei at time t. Two radioactive sources, S_1 and S_2 have same activity at time $t = 0$. At a later time, the activities of S_1 and S_2 are A_1 and A_2, respectively. When S_1 and S_2 have just completed their 3rd and 7th half-lives, respectively, the ratio $\frac{A_1}{A_2}$ is ________. **[4]**

15. In a particular system of units, a physical quantity can be expressed in terms of the electric charge e, electron mass m_e, Planck's constant h, and Coulomb's constant $k = \frac{1}{4\pi\epsilon_0}$, where ϵ_0 is the permittivity of vacuum. In terms of these physical constants, the dimension of the magnetic field is $[B] = [e]^{\alpha}[m_e]^{\beta}[h]^{\gamma}[k]^{\delta}$. The value of $\alpha + \beta + \gamma + \delta$ is ________. **[4]**

16. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius $\frac{R}{2}$ with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance 2R from the axis of the cylinder, is given by the expression $\frac{23pR}{16k\in_0}$. The value of k is: **[4]**



Chemistry

17. A solution of sodium sulphate in water is electrolysed using inert electrodes. The products at the cathode and anode are respectively **[3]**

a) O_2, Na
b) H_2, O_2
c) O_2, SO_2
d) O_2, H_2

18. When the same amount of zinc is treated separately with excess of sulphuric acid and excess of sodium hydroxide, the ratio of volumes of hydrogen evolved is **[3]**

a) 1 : 1

b) 9 : 4

c) 1 : 2

d) 2 : 1

19. For the chemical reaction, **[3]**

$3X(g) + Y(g) \rightleftharpoons X_3Y(g)$

the amount of X_3Y at equilibrium is affected by

a) temperature only

b) temperature and pressure

c) temperature, pressure and catalyst

d) pressure only

20. Which one of the following alkaline earth metal sulphates has its hydration enthalpy greater than its lattice enthalpy? **[3]**

a) $SrSO_4$

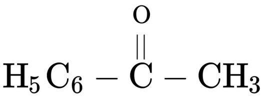
b) $BeSO_4$

c) $BaSO_4$

d) $CaSO_4$

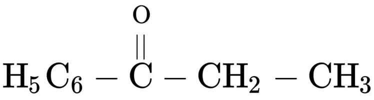
21. Keto-enol tautomerism is observed in **[4]**

a)



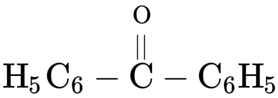
$H_5C_6 - \overset{\overset{O}{||}}{C} - CH_3$

b)



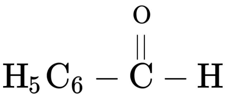
$H_5C_6 - \overset{\overset{O}{||}}{C} - CH_2 - CH_3$

c)



$H_5C_6 - \overset{\overset{O}{||}}{C} - C_6H_5$

d)



$H_5C_6 - \overset{\overset{O}{||}}{C} - H$

22. A plot of the number of neutrons (N) against the number of protons (P) of stable nuclei exhibits upward deviation from linearity for atomic number, $Z > 20$. For an unstable nucleus having $\frac{N}{P}$ ratio less than 1, the possible mode(s) of decay is (are) **[4]**

a) β^--decay (β emission)

b) orbital or K-electron capture

c) β^+-decay (positron emission)

d) neutron emission

23. An aqueous solution of hydrazine (N_2H_4) is electrochemically oxidized by O_2, thereby releasing chemical energy in the form of electrical energy. One of the products generated from the electrochemical reaction is $N_{2(g)}$. **[4]**

Choose the correct statement(s) about the above process.

a) At the cathode, N_2H_4 breaks to $N_{2(g)}$ and nascent hydrogen

b) Oxides of nitrogen are major by-products of the released at the electrode reacts with oxygen to form water.

electrochemical process.

c) At the cathode, molecular oxygen gets converted to OH^-.

d) OH^- ions react with N_2H_4 at the anode to form $N_{2(g)}$ and water, releasing 4 electrons to the anode.

24. The correct statement(s) related to the metal extraction processes is(are) **[4]**

a. A mixture of PbS and PbO undergoes self-reduction to produce Pb and SO_2

b. In die extraction process of copper from copper pyrites, silica is added to produce copper silicate.

c. Partial oxidation of sulphide ore of copper by roasting, followed by self-reduction produces blister copper.

d. In cyanide process, zinc powder is utilized to precipitate gold from Na[Main $Au(CN)_2$]

a) Statement (c) is correct.

b) Statement (a) is correct.

c) Statement (d) is correct.

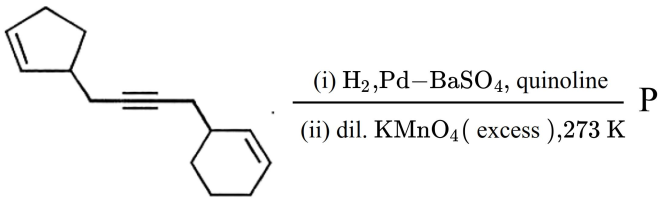
d) Statement (b) is correct.

25. Ozonolysis of ClO_2 produces an oxide of chlorine. The average oxidation state of chlorine in this oxide is ________. **[4]**

26. When the following aldohexose exists in its D-configuration, the total number of stereoisomers in its pyranose form is: **[4]**

$CHO - CH_2 - CHOH - CHOH - CHOH - CH_2OH$

27. Total number of hydroxyl groups present in a molecule of major product P is ________ **[4]**



28. For the octahedral complexes of Fe^{3+} in SCN^- (thiocyanato-S) and in CN^- ligand environments, the difference between the spin only magnetic moments in Bohr magnetons (when approximated to the nearest integer) is [atomic number of Fe = 26] **[4]**

29. A straight glass tube has two inlets X and Y at two ends. The length of the tube is 200 cm. HCl gas through inlets X and NH_3 gas through inlet Y are allowed to enter the tube at the same time. What fumes appear at point P inside the tube. Find the distance of P from X. **[4]**

30. The 1st, 2nd, and the 3rd ionization enthalpies, I_1, I_2, and I_3, of four atoms with atomic numbers n, n + 1, n + 2, and n + 3, where n < 10, are tabulated below. What is the value of n? **[4]**

Atomic number	**Ionization Enthalpy (kJ/mol)**		
	$\mathbf{I_1}$	$\mathbf{I_2}$	$\mathbf{I_3}$
n	1681	3374	6050
n + 1	2081	3952	6122
n + 2	496	4562	6910
n + 3	738	1451	7733

31. 20% of surface sites are occupied by N_2 molecules. The density of surface site is 6.023×10^{14} cm^{-2} and total surface area is 1000 cm^2. The catalyst is heated to 300 K while N_2 is completely desorbed into pressure of 0.001 atm and volume of 2.46 cm^3. Find the number of active sites occupied by each N_2 molecule. **[4]**

32. The total number of alkenes possible by dehydrobromination of 3-bromo-3-cyclopentylhexane using alcoholic KOH is **[4]**

Mathematics

33. The value of the expression ${}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$ is **[3]**

a) ${}^{52}C_5$ b) ${}^{47}C_5$

c) ${}^{52}C_4$ d) ${}^{52}C_{26}$

34. An infinite GP has first term **x** and sum 5, then x belongs to **[3]**

a) $-10 < x < 0$ b) $0 < x < 10$

c) $x > 10$ d) $x < -10$

35. Consider any set of 201 observations $x_1, x_2, \ldots x_{200}, + x_{201}$. It is given that $x_1 < x_2 < \ldots < x_{200} < x_{201}$. Then the mean deviation of this set of observations about a **[3]**

point k is minimum when k equals:

a) $\frac{(x_1+x_2+...+x_{200}+x_{201})}{201}$ b) x_{101}

c) x_1 d) x_{201}

36. The general value of θ satisfying the equation $2\sin^2\theta$ - $3\sin\theta$ - 2 = 0, is **[3]**

a) $n\pi + (-1)^n \frac{\pi}{6}$ b) $n\pi + (-1)^n \frac{\pi}{2}$

c) $n\pi + (-1)^n \frac{5\pi}{6}$ d) $n\pi + (-1)^n \frac{7\pi}{6}$

37. Let f: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \rightarrow$ R be given by f(x) = $(\log(\sec x + \tan x))^3$. Then **[4]**

a) f(x) is an odd function b) f(x) is an onto function

c) f(x) is one-one function d) f(x) is an even function

38. If $\alpha = 3\sin^{-1}\left(\frac{6}{11}\right)$ and $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$, where the in inverse trigonometric functions take only the principal values, then the correct option(s) is (are) **[4]**

a) $\cos\beta > 0$ b) $\sin\beta < 0$

c) $\cos(\alpha + \beta) > 0$ d) $\cos\alpha < 0$

39. If $3^x = 4^{x-1}$, then x = **[4]**

a) $\frac{1}{1-\log_4 3}$ b) $\frac{2\log_2 3}{2\log_2 3-1}$

c) $\frac{2\log_3 2}{2\log_3 2-1}$ d) $\frac{2}{2-\log_2 3}$

40. Let E, F and G be three events having probabilities P(E) = $\frac{1}{8}$, P(F) = $\frac{1}{6}$ and P(G) = $\frac{1}{4}$, and let P(E ∩ F ∩ G) = $\frac{1}{10}$ **[4]**

For any event H, if H^c denotes its complement, then which of the following statements is(are) TRUE?

a) $P(E^C F^C G^C)$ b) $P(E^C \cap F \cap G) \leq \frac{1}{15}$

c) $P(E \cap F \cap G^C) \leq \frac{1}{40}$ d) $P(E \cup F \cup G) \leq \frac{13}{24}$

41. Let ω be the complex number $\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$. Then the number of distinct complex numbers z satisfying $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}$ = 0 is equal to **[4]**

42. Let ABC and ABC' be two non-congruent triangles with sides AB = 4, AC = AC' = $2\sqrt{2}$ and angle $B = 30^\circ$. The absolute value of the difference between the areas of these triangles is **[4]**

43. Let f: R $\to$ R and g: R $\to$ R be respectively given by f(x) = $|x| + 1$ and g(x) = x^2 + 1. Define h: R $\to$ R by **[4]**

$$h(x) = \begin{cases} \max\{f(x), g(x)\}, & \text{if } x \leq 0. \\ \min\{f(x), g(x)\}, & \text{if } x > 0. \end{cases}$$

The number of points at which h(x) is not differentiable is

44. Consider the region R = {(x ,y) $\in$ R x R : x $\geq$ 0 and $y^2 \leq$ 4-x}. **[4]**

Let F be the family of all circles that are contained in R and have centers on the x-axis. Let C be the circle that has largest radius among the circles in F. Let (α, β) be a point where the circle C meets the curves y^2 = 4 - x. The value of α is ________.

45. Let f : $\mathbb{R} \to \mathbb{R}$ be a differentiable function with f(0) = 1 and satisfying the equation f(x + y) = f(x) f'(y) + f'(x) f(y) for all x, y $\in \mathbb{R}$. Then, the value of $\log_e$ (f(4)) is ________. **[4]**

46. Let p(x) be a polynomial of degree 4 having extremum at x = 1, 2 and $\lim_{x\to 0}\left(1 + \frac{p(x)}{x^2}\right)$ = 2. Then the value of p(2) is **[4]**

47. Let P be the plane $\sqrt{3}x$ + 2y + 3z = 16 and let **[4]**

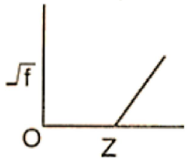
$S = \left\{\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k} : \alpha^2 + \beta^2 + \gamma^2 = 1\right.$ and the distance of (α, β, γ) from the plane P is $\left.\frac{7}{2}\right\}$. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three distinct vectors in S such that $|\vec{u} - \vec{v}| = |\vec{v} - \vec{w}| = |\vec{w} - \vec{u}|$. Let V be the volume of the parallelepiped determined by vectors $\vec{u}, \vec{v}$ and $\vec{w}$. Then the value of $\frac{80}{\sqrt{3}}V$ is

48. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a} = \frac{\hat{\mathrm{i}} - 2\hat{\mathrm{j}}}{\sqrt{5}}$ and $\vec{b} = \frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$, then the value of $(2\vec{a} + \vec{b}) \cdot [(\vec{a} \times \vec{b}) \times (\vec{a} - 2\vec{b})]$ is **[4]**

Solution

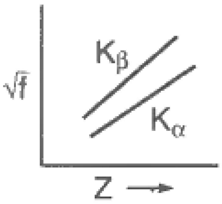
Physics

1.

(c) 

Explanation:

Moseley studied the X-ray spectra of various elements. The spectral line observed were of



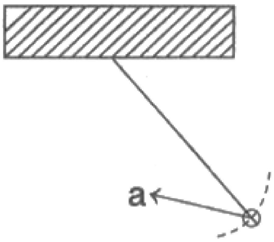
i. short wavelength X-series and

ii. long-wavelength L-series. K_α line is most intense in the X-series. Moreover, he observed that the wavelength of the K_α line decreases with increase in the atomic number of the element as the target. If a graph is plotted between the square root of the frequency and the atomic number of the element emitting the line, it is a straight line.

Thus, $\sqrt{f} \propto Z$

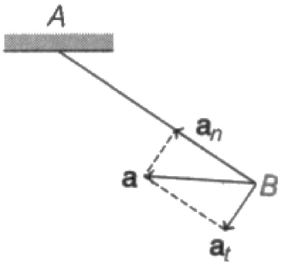
Where, $\sqrt{f}$ is the frequency of the radiation and Z is the atomic number of the element.

2.

(b) 

Explanation:

Net acceleration **a** of the bob in position B has two components.



a. a_n = radial acceleration (towards BA)

b. a_t = tangential acceleration (perpendicular to BA)

Therefore, direction of **a** is correctly shown in option (c).

3\.

(c) $E^{1/2}$

Explanation:

$$\frac{\lambda_1}{\lambda_2} = \frac{\frac{h}{\sqrt{2mE}}}{\frac{hc}{E}} \text{ or } \frac{\lambda_1}{\lambda_2} \propto E^{1/2}$$

4\.

(b) $\tan^{-1}(4)$

Explanation:

$H = \frac{u^2 \sin^2 \theta}{2g}$ and $R = \frac{u^2 \sin 2\theta}{g}$

Since, H = R

$$\frac{u^2 \sin^2 \theta}{2g} = \frac{u^2 \times 2 \sin \theta \cos \theta}{g}$$

or tanθ = 4 or θ = tan^{-1}(4).

5\. **(a)** The vertical component of reaction force at O does not depend on α.

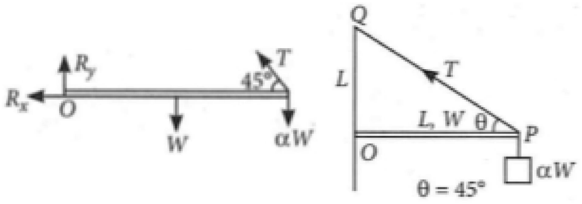
(c) The rope breaks if $\alpha > 1.5$

(d) The horizontal component of reaction force at O is equal to W for $\alpha = 0.5$

Explanation: Max. tension in rope $T = 2\sqrt{2}W$

Free body diagram of rope is shown.

Balance the forces along x and y-axis



$R_x = T\cos 45^\circ = \frac{T}{\sqrt{2}}$ ...(i)

$R_y + T\sin 45^\circ = W + \alpha W;\ R_y + \frac{T}{\sqrt{2}} = W + \alpha W$ ...(ii)

Take the moments about O.

$W \times \frac{l}{2} + \alpha W L = T\sin 45^\circ \times L; T = \sqrt{2}\left(\frac{W}{2} + \alpha W\right)$ ...(iii)

Take the moments about P

$R_y L = W\frac{L}{2} \Rightarrow R_y = \frac{W}{2}$

When tension is maximum, $T = T_{\text{max}}$

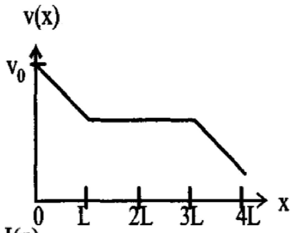
From equation (iii), $2\sqrt{2}W = \sqrt{2}\left(\frac{W}{2} + \alpha W\right)$

$\Rightarrow 2\sqrt{2}W = W\sqrt{2}\left(\frac{1}{2} + \alpha\right) \Rightarrow 2 - \frac{1}{2} = \alpha \Rightarrow \alpha = \frac{3}{2}$

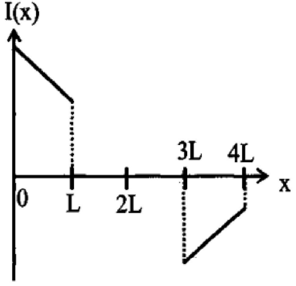
$R_x = \frac{T}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \sqrt{2}\left(\frac{W}{2} + \alpha W\right)$

If $\alpha = 0.5; R_x = \frac{W}{2} + 0.5W = W$

6. **(b)**



(d)



Explanation: $i = \frac{e}{R} = \frac{BLv}{R}$ ...(i) [Counter-clockwise direction while entering, Zero when completely inside and clockwise while exiting]

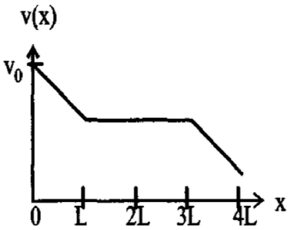
$F = iLB = \frac{B^2L^2v}{R}$ (ii) [Toward left while entering and exiting and zero when completely inside]

$\therefore -mV\frac{dv}{dx} = \frac{B^2L^2v}{R}$

$\therefore \int_{V_0}^{V} dV = -\frac{B^2L^2}{mR}\int_0^x dx \Rightarrow V - V_0 = -\frac{B^2L^2}{mR}x$

$\therefore V = V_0 - \frac{B^2L^2x}{mR}$ ...(iii)

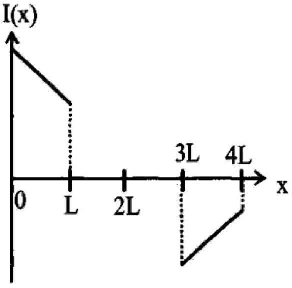
[V decreases from x = 0 to x = L, remains constant for x = L to x = 3L again decreases from x = 3L to x = 4 L hence given below graph is correct]



From (i) and (iii)

$$i = \frac{BL}{R}\left[V_0 - \frac{B^2L^2x}{mR}\right]$$

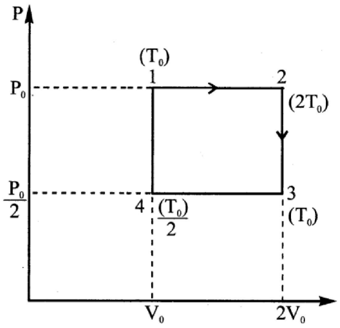
[i decreases from x = 0 to x = Li becomes zero from x = L to x = 3Li changes direction and decreases from x = 3L to x = 4L] Hence graph given below is correct.



7. **(a)** Work done in this thermodynamic cycle ($1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1$) is $|W| = \frac{1}{2}RT_0$

(d) The ratio of heat transfer during processes $1 \rightarrow 2$ and $3 \rightarrow 4$ is $\left|\frac{Q_{1\rightarrow 2}}{Q_{2\rightarrow 3}}\right| = \frac{5}{3}$

Explanation: The P-V graph of the given V-T graph is given below.



- Work done during cyclic process $(1 \rightarrow 2 \rightarrow 3 \rightarrow 4 - 1)$W = are enclosed in the loop

  $= \frac{P_0}{2} V_0$

  $\because P_0 V_0 = nRT_0 \therefore \frac{P_0 V_0}{2} = \frac{nRT_0}{2}$

  $\therefore$ Work done W = $\frac{nRT_0}{1} = \frac{RT_0}{2}$ [as n = 1]

- Process $1 \rightarrow 2$ is isobaric

  Process $2 \rightarrow 3$ is isochoric

  Process $3 \rightarrow 4$ is isobaric

  Process $4 \rightarrow 1$ is isochoric

  Hence no adiabatic process is involved.

- $|\Delta Q_{1 \rightarrow 2}| = |nC_p \Delta T| = \left|nC_p \left(2\,T_0 - T_0\right)\right| = |nC_p T_0|$

  $|\Delta Q_{2 \rightarrow 3}| = |\Delta U| = |nC_v \Delta T| = |nC_v T_0|$

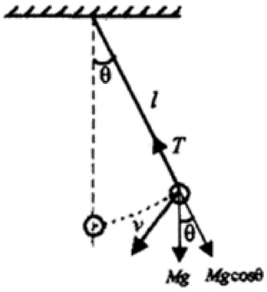
  $\therefore \left|\frac{\Delta Q_{1\rightarrow 2}}{\Delta Q_{2\rightarrow 3}}\right| = \frac{C_p}{C_v} = \frac{5}{3}$

- $|\Delta Q_{3 \rightarrow 4}| = nC_p \frac{T_0}{2} \therefore \left|\frac{\Delta Q_{1\rightarrow 2}}{\Delta Q_{3\rightarrow 4}}\right| = \frac{nC_p T_0}{nC_p \frac{T_0}{2}} = \frac{2}{1}$

8. **(a)** The magnitude of the tangential acceleration of the bob $|a_T| = \text{g} \sin \theta$

   (b) 7 - Mgcos$\theta = \frac{MV^2}{L}$

   Explanation: A long radius net force = centripetal force $\left(\frac{Mv^2}{\ell}\right)$



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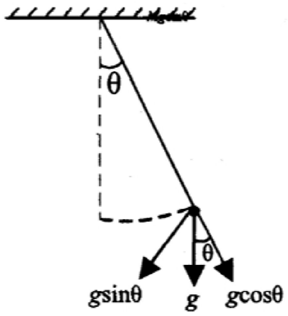
And along tangent net force = ma_t as the motion of a pendulum is the part of circular motion.



$\therefore T - Mg\cos\theta = \frac{Mv^2}{\ell}$

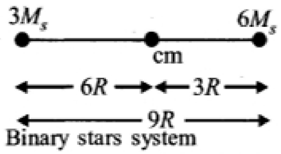
And, $ma_t = mg\sin\theta \Rightarrow a_t = g\sin\theta$

9. 9.0

Explanation:

The centre of mass lies at a distance 6R from lighter mass In circular orbit

Time period, $T = 2\pi\sqrt{\frac{R^3}{GM_S}}$



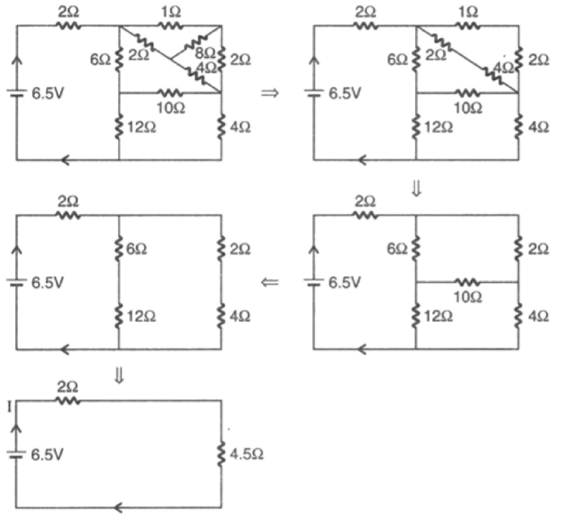
$$nT = 2\pi\sqrt{\frac{(9R)^3}{G(3M_S+6M_S)}}$$

$$\text{or, } n \times 2\pi\sqrt{\frac{R^3}{GM_S}} = 9 \times 2\pi\sqrt{\frac{R^3}{GM_S}}$$

$$\left[\because T = 2\pi\sqrt{\frac{R^3}{GM_S}}\right] \therefore n = 9$$

10. 1

Explanation:



$I = \frac{6.5}{6.5} = 1\text{amp}$

11. 47.10

Explanation:

Since, pressure P = constant $\rho_a T_a = \rho T$

$\Rightarrow 1.2 \times 300 = \rho(360) \therefore \rho = 1\ \text{kg/m}^3$

Applying Bernoullis theorem between upper and bottom point

Assuming velocity of hot air inside the furnace $\simeq 0$

$P_a + 0 + 0 = P_a - \rho_a g(h) + \rho g(h) + \frac{1}{2}\rho V^2$

$\therefore V = \sqrt{\frac{2(\rho_a - \rho)g \times 9}{\rho}} = \sqrt{2(0.2)90} = 6$

Therefore the steady mass flow rate of air existing the chimney

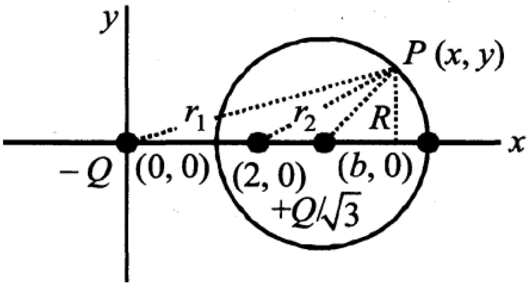
$Q = \rho\pi\left(\frac{d^2}{4}\right)V = 1 \times 3.14\,\frac{\times(0.1)^2}{4} \times 6$

$= 0.0471$ kg/s = 47.10 gms^{-1}

12. 1.73

Explanation:

let us consider a point P on the circle



$V_P = 0 = \frac{k(-Q)}{r_1} + \frac{\frac{kQ}{\sqrt{3}}}{r_2} \Rightarrow \frac{kQ}{r_1} = \frac{\frac{kQ}{\sqrt{3}}}{r_2}$

$\Rightarrow \frac{1}{\sqrt{x^2+y^2}} - \frac{1}{\sqrt{3}\sqrt{(x-2)^2+y^2}}$

$\Rightarrow 3(x-2)^2 + 3y^2 = x^2 + y^2$

$\Rightarrow 3\left(x^2 + 4 - 4x\right) - x^2 + 2y^2 = 0 \Rightarrow 2x^2 + 12 - 12x + 2y^2 = 0$

$\Rightarrow x^2 + 6 - 6x + y^2 = 0 \Rightarrow (x-3)^2 + y^2 = (\sqrt{3})^2$

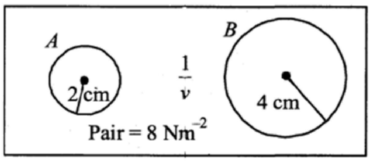
or $(x-b)^2 + y^2 = (\sqrt{3})^2 = R^2$

$\therefore R = \sqrt{3} = 1.73$ and $b = 3$

13. 6

Explanation:

Although not given in the question, but we will have to assume that temperatures of A and B are the same.



$\frac{n_B}{n_A} = \frac{p_B V_B / RT}{p_A V_A / RT} = \frac{p_B V_B}{p_A V_A}$

$= \frac{p + 4S/r_A \times 4/3\pi(r_A)^3}{(p + 4S/r_B) \times 4/3\pi(r_B)^3}$ (S = surface tension)

Substituting the values, we get,

$\frac{n_B}{n_A} = 6$

14. 16.0

Explanation:

When radioactive sources just completed their 3^{rd} and 7^{th} half - lives then the ratio

$\frac{A_1}{A_2} = \frac{A_0 e{-}3 \ln 2}{A_0 e{-}7\ell n2} = \frac{2^{-3}}{2^{-7}} = \frac{1}{2^{(-4)}} = 2^4 = 16$

15. 4

Explanation:

$[\mathrm{B}] = [\mathrm{e}]^\alpha [\mathrm{m_e}]^\beta [\mathrm{h}]^\gamma [\mathrm{k}]^\delta$

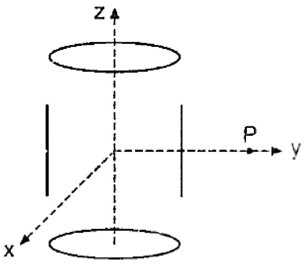
$\Rightarrow \mathrm{MT}^{-2}\ \mathrm{A}^{-1} = \mathrm{A}^\alpha \mathrm{T}^\alpha \mathrm{M}^\beta \mathrm{M}^\gamma \mathrm{L}^{2\gamma} \mathrm{T}^{-\gamma} \mathrm{M}^\delta \mathrm{L}^{3\delta} \mathrm{A}^{-2\delta} \mathrm{T}^{-4\delta}$

$\Rightarrow \beta + \gamma + \delta = 1, -2 = \alpha - \gamma - 4\delta, -1 = \alpha - 2\delta\ 2\gamma + 3\delta = 0$

$\Rightarrow \alpha = 3, \beta = 2, \gamma = -3, \delta = 2 \Rightarrow \alpha + \beta + \gamma + \delta = 4$

16. 6

Explanation:



$\vec{E} = \frac{\lambda(\hat{j})}{2\pi\varepsilon_0(2R)} + \frac{K\left(\rho \frac{4}{3}\pi \frac{R^3}{8}\right)(-\hat{j})}{4R^2}$

$= \frac{\rho\pi R^2(\hat{j})}{4\pi\varepsilon_0 R} + \frac{K\pi\rho R(-\hat{j})}{24}$

$= K\rho\pi R(j) + \frac{K}{24}\rho\pi R(-\hat{j})$

$= k\rho\pi R\frac{23}{24}(\hat{j}) = \frac{23}{96\varepsilon} - \rho R(\hat{j})$

$\therefore$ K = 6

Chemistry

17.

(b) H_2, O_2

Explanation:

Water is reduced at the cathode and oxidized at the anode instead of Na^+ and SO_4^{2-}

Cathode: $2H_2O + 2e^- \rightarrow H_2 + 2OH^-$; E° = -0.83 V

Anode: $H_2O \rightarrow 2H^+ + \frac{1}{2}O_2 + 2e^-$; E^O = -1.23 V

Note: The standard electrode, reduction potential of Na^+ is less than that of water.

$Na^+ + e^- \rightarrow \underset{oxidation}{Na(s)}$ E^O = -2.71V

The standard electrode, oxidation potential of SO_4^{2-} is less than that of water.

$2SO_4^{2-} \longrightarrow S_2O_8^{2-} + 2e^-$ E^O = -2.01V

18. **(a)** 1 : 1

Explanation:

The balanced chemical reaction of zinc with sulphuric acid and NaOH are

$Zn + H_2SO_4 \longrightarrow ZnSO_4 + H_2(g) \uparrow$

$Zn + 2NaOH + 2H_2O \longrightarrow Na_2[Zn(OH)_4] + H_2(g) \uparrow$

Since, one mole of $H_2(g)$ is produced per mole of zinc with both sulphuric acid and NaOH respectively, hydrogen gas is produced in the molar ratio of 1:1 in the above reactions.

19.

(b) temperature and pressure

Explanation:

The given reaction will be exothermic in nature due to the formation of three X - Y bonds from the gaseous atoms. The reaction is also accompanied with the decrease in the gaseous species (i.e. Δn is negative). Hence, the reaction will be affected by both temperature and pressure. The use of catalyst does not affect the equilibrium concentrations of the species in the chemical reaction.

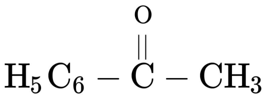
20.

(b) $BeSO_4$

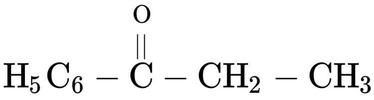
Explanation:

As we move down the group, the size of metal increases. Be has lower size while SO_4^{2-} has a bigger size, that's why $BeSO_4$ breaks easily and lattice energy becomes smaller but due to lower size of Be, water molecules are gathered around and hence hydration energy increases. On the other hand, the rest of the metals, i.e. Ca, Ba, Sr have a bigger size and that's why lattice energy is greater than hydration energy.

The time-saving technique in the question of finding hydration energy only checks the size of the atom. Smaller sized atom has more hydration energy. Thus, in this question Be is placed uppermost in the group has a lesser size and not comparable with the size of sulphates. Hence, $BeSO_4$ is the right response.

21. **(a)** 

$H_5C_6 - \overset{\overset{O}{\|}}{C} - CH_3$

(b) 

$H_5C_6 - \overset{\overset{O}{\|}}{C} - CH_2 - CH_3$

Explanation: Keto-enol tautomerism is shown in compounds having α-hydrogen on the C adjacent to the CO group.

22. **(b)** orbital or K-electron capture

(c) β^+-decay (positron emission)

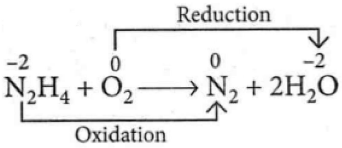
Explanation: When $\frac{N}{P}$ ratio is less than one, then proton changes into neutron to increase the ratio.

Position emission: ${}_1^1H \longrightarrow {}_0^1n + {}_{+1}^{0}\beta$

K-electron capture: ${}_1^1H + {}_{-1}^{0}e \longrightarrow {}_0^1n + \text{X-rays}$

23. **(c)** At the cathode, molecular oxygen gets converted to OH^-.

(d) OH^- ions react with N_2H_4 at the anode to form $N_{2(g)}$ and water, releasing 4 electrons to the anode.

Explanation: 

$$\overset{-2}{N}_2H_4 + \overset{0}{O}_2 \longrightarrow \overset{0}{N}_2 + 2H_2\overset{-2}{O}$$

At anode: $N_2H_4 + 4OH^- \longrightarrow N_2 + 4H_2O + 4e^-$

At cathode: $O_2 + 2H_2O + 4e^- \longrightarrow 4OH^-$

Overall reaction: $N_2H_4 + O_2 \longrightarrow N_2 + 2H_2O$

So, statements (OH^- ions react with N_2H_4 at the anode to form $N_{2(g)}$ and water, releasing 4 electrons to the anode.) and (At the cathode, molecular oxygen gets converted to OH^-.) are correct.

24. **(a)** Statement (c) is correct.

(b) Statement (a) is correct.

(c) Statement (d) is correct.

Explanation:

a. $PbS + 2PbO \rightarrow 3Pb + SO_2$ (self reduction)

b. Silica is added to remove impurity of Fe in the form of slag $FeSiO_3$. Hence, this statement is wrong.

c. Sulphide ore is partially oxidized first by roasting and then self-reduction of Cu takes place to produce blister cop.

$Cu_2\,S + \frac{3}{2}O_2 \rightarrow Cu_2O + SO_2 \uparrow$

$Cu_2\,S + 2Cu_2O \rightarrow 6Cu + SO_2 \uparrow$ (self reduction)

The molten copper obtained is poured into large container and allowed to cool and during cooling the dissolved SO_2 comes up to the surface and forms blisters. It is known as blister copper.

d. $$2Na\left[\overset{+1}{Au}(CN)_2\right] + \underset{\text{Reducing Agent}}{Zn} \rightarrow Na_2\left[\overset{+2}{Zn}(CN)_4\right] + 2\overset{0}{Au}$$

25. 6.0

Explanation:

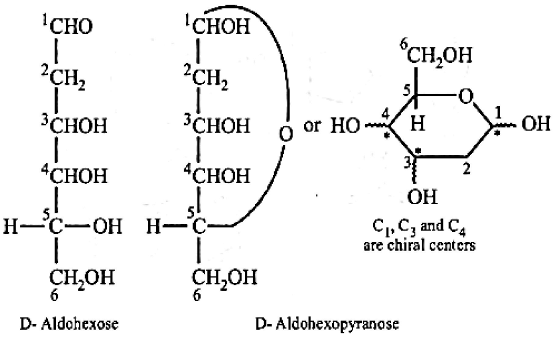
$2ClO_2 + 2O_3 \rightarrow Cl_2O_6 + 2O_2$

$Cl_2O_6 \Rightarrow 2x + 6(-2) = 0$

$x = +6$

26. 8

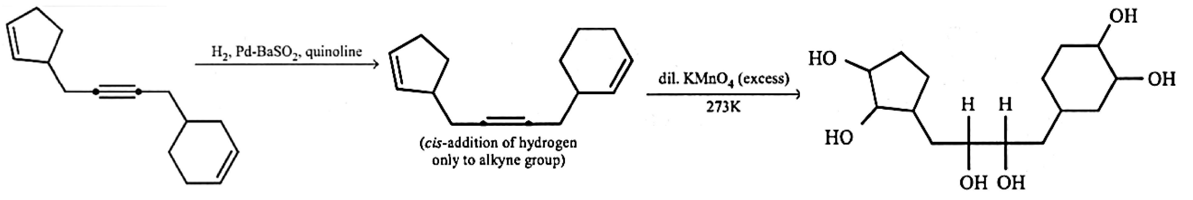
Explanation:



Thus, total number of stereoisomers in pyranose form of D-configuration = $2^3 = 8$.

27. 6

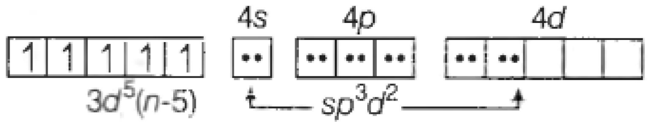
Explanation:



28. 4

Explanation:

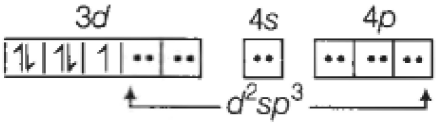
When S is donor atom of SCN-, it produces a weak ligand field $[Fe(SCN)_6]^{3-}$: Fe^3 + $(3d^5)$ =



Spin only magnetic moment $(\mu_s) = \sqrt{5(5+2)}\text{BM} = \sqrt{35}\text{BM}$

In the case of CN“ ligand, carbon is the donor atom, it produces a strong ligand field and forms a low spin complex as

$Fe(CN_6)^3$: $Fe^{3+}(3d^5)$



Spin only magnetic moment $(\mu_s) = \sqrt{1(1+2)}\text{BM} = \sqrt{3}\text{BM}$

Hence, the difference in spin only magnetic moment

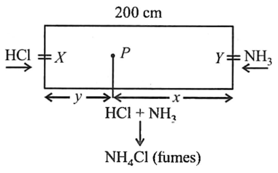
$= \sqrt{35} - \sqrt{3} \approx 4\text{BM}$

29. 85.2

Explanation:

Let NH_3 diffuse through = x cm

HCl diffuses through = y cm



According to Graham’s law of diffusion

$$\frac{x}{y} = \sqrt{\frac{\text{Mol. wt } HCl}{\text{Mol. wt of } NH_3}} = \sqrt{\frac{36.5}{17}} = \sqrt{2.14} = 1.465$$

x = 1.465 y ...(i)

x + y = 200 cm ...(ii)

From these equations; y = 85.2 cm

Distance between P and X = y = 85.2 cm

30. 9

Explanation:

By observing the values of ionization enthalpy for atomic number (n + 2), it is observed that $I_2 >> I_1$. This shows that a number of valence shell electrons is 1 for atomic number (n + 2). Therefore element with an atomic number (n + 2) should be an alkali metal.

For atomic number (n + 3), $I_3 >> I_2$, which shows that it will be an alkaline earth metal.

All the observations suggest that atomic number (n + 1) should be a noble gas and atomic number (n) should belong to the halogen family. Since n < 10; hence n = 9.

31. 2

Explanation:

$P_{N_3} = 0.001$ at, T = 300 K, V = 2.46 cm^2

$\therefore$ Number of N_2 molecules

$= \frac{PV}{RT} \times N_A = \frac{0.001 \times 2.46 \times 10^{-3}}{0.0821 \times 300} \times 6.023 \times 10^{23}$

$= 6.016 \times 10^{16}$

Now, the total number of surface sites = Density × Total surface area

$= 6.023 \times 10^{14} \times 1000 = 6.023 \times 10^{17}$

Sites occupied by N_2 molecules $= \frac{20}{100} \times 6.023 \times 10^{17}$

$= 12.04 \times 10^{16}$

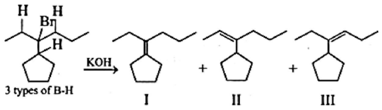
$\therefore$ No. of sites occupied by each N_2 molecule

$= \frac{12.04 \times 10^{16}}{6.016 \times 10^{16}} \approx 2$

32. 5

Explanation:

The substrate has three different types of B—H, therefore, first three structural isomers of alkenes are expected as:



The last two alkenes II and III are also capable of showing geometrical isomerism, hence two geometrical isomers for each of them will be counted giving a total of five isomers.

Mathematics

33.

(c) $^{52}C_4$

Explanation:

Here, $^{47}C_4+\sum_{j=1}^5{ }^{52-j} C_3$

$={ }^{47} C_4+{ }^{51} C_3+{ }^{50} C_3+{ }^{49} C_3+{ }^{48} C_3+{ }^{47} C_3$

$=\left({ }^{47} C_4+{ }^{47} C_3\right)+{ }^{48} C_3+{ }^{49} C_3+{ }^{50} C_3+{ }^{51} C_3$

[using $^n C_r+{ }^n C_{r-1}={ }^{n+1} C_r$]

$=\left({ }^{48} C_4+{ }^{48} C_3\right)+{ }^{49} C_3+{ }^{50} C_3+{ }^{51} C_3$

$=\left({ }^{49} C_4+{ }^{49} C_3\right)+{ }^{50} C_3+{ }^{51} C_3$

$=\left({ }^{50} C_4+{ }^{50} C_3\right)+{ }^{51} C_3$

$={ }^{51} C_4+{ }^{51} C_3={ }^{52} C_4$

34.

(b) $0 < x < 10$

Explanation:

We know that, the sum of infinite terms of GP is

$S_\infty=\begin{cases}\frac{a}{1-r},|r|<1 \\ \infty,|r| \geq 1\end{cases}$

$\therefore S_\infty = \frac{x}{1-r} = 5$ [| r | < 1]

or 1 - r = $\frac{x}{5}$

$\Rightarrow$ r = $\frac{5-x}{5}$ exists only when | r | <1.

i.e. - 1 < $\frac{5-x}{5}$ < 1

or -10 < - x < 0

$\Rightarrow$ 0 < x < 10

35.

(b) x_{101}

Explanation:

Given, that $x_1 < x_2 < x_3 < ... < x_{201}$

$\therefore$ Median of the given obswervation = $\frac{201+1}{2}$th obs.

= 101^{th} obs. = x_{101}

We know that deviations will be minimum if taken from the median

$\therefore$ Mean deviation will be minimum if k = x_{101}

36.

(d) $n \pi+(-1)^n \frac{7 \pi}{6}$

Explanation:

Given, 2 $\sin^2 \theta$ - 3 $\sin \theta$ - 2 = 0

$\Rightarrow$ (2 $\sin \theta$ + 1) ($\sin \theta$ - 2) = 0

$\Rightarrow \sin\theta = \frac{-1}{2}$ [neglecting $\sin\theta = 2$, as $|\sin\theta| \le 1$]

$\therefore \theta = n\pi + (-1)^n(7\pi/6)$

37. **(a)** f(x) is an odd function

(b) f(x) is an onto function

(c) f(x) is one-one function

Explanation: Given: f: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to$ R is given by

$f(x) = (\log(\sec x + \tan x))^3$

$f(-x) = (\log(\sec x - \tan x))^3$

$= \left[\log\left(\frac{(\sec x - \tan x)(\sec x + \tan x)}{\sec x + \tan x}\right)\right]^3$

$= \left[\log\left(\frac{1}{\sec x + \tan x}\right)\right]^3 = [-\log(\sec x + \tan x)]^3$

$= -[\log(\sec x + \tan x)]^3 = -f(x)$

$\therefore$ f(x) is an odd function.

$\therefore$ option (f(x) is an odd function) is correct and (f(x) is an even function) is not correct.

Now, $f'(x) = 3[\log(\sec x + \tan x)]^2 . \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$

$= 3 \sec x[\log(\sec x + \tan x)]^2 > 0 \ \forall\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore$ f(x) is increasing on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

We know that strictly increasing function is one one.

$\therefore$ f is one one, hence (f(x) is one-one function) is the correct option.

Also $\lim\limits_{x \to \frac{\pi^-}{2}} [\log(\sec x + \tan x)]^3 \to \infty$

and $\lim\limits_{x \to \frac{\pi^+}{2}} [\log(\sec x + \tan x)]^3 \to -\infty$

$\therefore$ Range of f = $(-\infty, \infty)$ = R = Domain

$\therefore$ f is an onto function.

$\therefore$ option (f(x) is an onto function) is correct.

38. **(b)** $\sin\beta < 0$

(c) $\cos(\alpha + \beta) > 0$

(d) $\cos\alpha < 0$

Explanation: $\alpha = 3\sin^{-1}\frac{6}{11} > 3\sin^{-1}\frac{1}{2} = \frac{\pi}{2} \Rightarrow \alpha > \frac{\pi}{2}$

$\therefore \cos\alpha < 0$

$\beta = 3\cos^{-1}\frac{4}{9} > 3\cos^{-1}\frac{1}{2} = \pi \Rightarrow \beta > \pi$

$\therefore \cos\beta < 0$ and $\sin\beta < 0$

Now $\alpha + \beta > \frac{3\pi}{2}, \therefore \cos(\alpha + \beta) > 0$

39. **(a)** $\frac{1}{1-\log_4 3}$

(c) $\frac{2\log_3 2}{2\log_3 2-1}$

(d) $\frac{2}{2-\log_2 3}$

Explanation: $3^x = 4^{x-1} \Rightarrow x \log 3 = 2(x-1)\log 2$

$\Rightarrow x = \frac{2 \log 2}{2 \log 2 - \log 3}$

$\Rightarrow x = \frac{2 \log_3 2}{2 \log_3 2 - 1} = \frac{2}{2-\log_2 3}$

Also, $x = \frac{1}{1-\frac{1}{2}\log_2 3} = \frac{1}{1-\log_4 3}$

40. **(b)** $P(E^C \cap F \cap G) \leq \frac{1}{15}$

(c) $P(E \cap F \cap G^C) \leq \frac{1}{40}$

(d) $P(E \cup F \cup G) \leq \frac{13}{24}$

Explanation: Given that

$P(E) = \frac{1}{8}$; $P(F) = \frac{1}{6}$; $P(G) = \frac{1}{4}$; $P(E \cap F \cap G) = \frac{1}{10}$

i. $P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(F \cap G) - P(G \cap E) + P(E \cap F \cap G)$

$= \frac{1}{8} + \frac{1}{6} + \frac{1}{4} - \sum P(E \cap F) + \frac{1}{10}$

$= \frac{13}{24} + \frac{1}{10} - \sum P(E \cap F)$

$\Rightarrow P(E \cup F \cup G) \leq \frac{13}{24}$ is correct option.

ii. Now, $P(E^C \cap F^C \cap G^C)$

$= 1 - P(E \cup F \cup G) \geq 1 - \frac{13}{24}$

$\Rightarrow P(E^C \cap F^C \cap G^C) \geq \frac{11}{24}$ is incorrect option.

iii. $\because P(E) \geq P(E \cap F \cap G^C) + P(E \cap F \cap G)$

$\Rightarrow \frac{1}{8} \geq P(E \cap F \cap G^C) + \frac{1}{10}$

$\Rightarrow \frac{1}{8} - \frac{1}{10} \geq P(E \cap F \cap G^C)$

$\Rightarrow \frac{1}{40} \geq P(E \cap F \cap G^C)$ is correct option.

iv. $\because P(F) \geq P(E^C \cap F \cap G) + P(E \cap F \cap G)$

$\Rightarrow \frac{1}{6} \geq P(E^C \cap F \cap G) + \frac{1}{10}$

$\Rightarrow \frac{1}{6} - \frac{1}{10} \geq P(E^C \cap F \cap G)$

$\Rightarrow \frac{4}{60} \geq P(E^C \cap F \cap G)$

$\Rightarrow \frac{1}{15} \geq P(E^C \cap F \cap G)$ is correct option.

41. 0

Explanation:

Given: $\omega = \cos\frac{2\pi}{3} + i \sin\frac{2\pi}{3} = \frac{-1+i\sqrt{3}}{2}$

$\therefore 1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$

Then $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} z+1+\omega+\omega^2 & \omega & \omega^2 \\ z+1+\omega+\omega^2 & z+\omega^2 & 1 \\ z+1+\omega+\omega^2 & 1 & z+\omega \end{vmatrix} = 0$

$[C_1 \rightarrow C_1 + C_2 + C_3]$

$$\Rightarrow z\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & z+\omega^2 & 1 \\ 1 & 1 & z+\omega \end{vmatrix} = 0$$

$\Rightarrow z[z^2] = 0 \Rightarrow z^3 = 0 \Rightarrow z = 0$

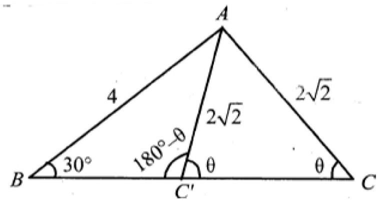
$\therefore$ z = 0 is the only solution.

42. 4.0

Explanation:

Let $\angle ACC' = \theta$, then $\angle AC'C = \theta \quad (\because AC = AC')$

and $\angle AC'B = 180^\circ - \theta$



Applying sine law in $\triangle ABC'$, we get

$\frac{4}{\sin(180-\theta)} = \frac{2\sqrt{2}}{\sin 30^\circ} \Rightarrow \sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$

$\therefore \angle CAC' = 90^\circ$

Now required area $= \text{ar}(\triangle ABC) - \text{ar}(\triangle ABC')$

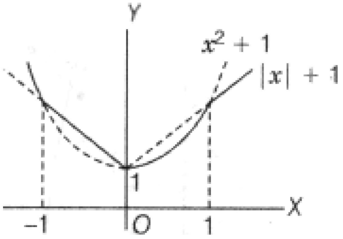
$= \text{ar}(\triangle ACC') = \frac{1}{2} \times AC \times AC'$

$= \frac{1}{2} \times 2\sqrt{2} \times 2\sqrt{2} = 4$ sq. units.

43. 3

Explanation:

Curve of f(x) and g(x) are



h(x) is not differentiable at $x = \pm 1$ and 0.

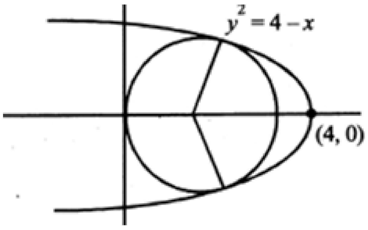
As, h(x) take sharp turns at $x = \pm 1$ and 0.

Hence, number of points of non-differentiability of h (x) is 3.

44. 2.0

Explanation:

Since C be the circle that has largest radius so, it touches the y-axis at (0, 0) and centre at x-axis.



Let the equation of circle be

$x^2 + y^2 + \lambda x = 0$

$\Rightarrow x^2 + x(\lambda - 1) + 4 = 0$ ...(i)

For point of intersection.

Put λ = -3 in equation (i)

$x^2 - 4x + 4 = 0$

$\Rightarrow x = 2$ so $\alpha = 2$

45. 2

Explanation:

$f(x + y) = f(x)\,f'(y) + f'(x)\,f(y)$ ...(i)

On putting x = y = 0, we get

$f(0) = 2f'(0)\,f(0) \Rightarrow f'(0) = \frac{1}{2}$ [$\because$ f(0) = 1]

On putting y = 0 in equation (i), we get

$f(x) = f(x)\,f'(0) + f'(x)\,f(0)$

$\Rightarrow f'(x) = \frac{f(x)}{2} \Rightarrow \int \frac{f'(x)}{f(x)}dx = \frac{1}{2}\int dx$ [$\because$ f(0) = 1 and f'(0) = $\frac{1}{2}$]

$\Rightarrow \log_e f(x) = \frac{x}{2} + \log_e c$

$\Rightarrow f(x) = ce^{x/2} \Rightarrow f(x) = e^{x/2}$

$\Rightarrow \log_e (f(x)) = \frac{x}{2} \Rightarrow \log_e (f(4)) = 2$

46. 0

Explanation:

Let $p(x) = ax^4 + bx^3 + cx^2 + dx + e$

Now $\lim\limits_{x \to 0}\left[1 + \frac{p(x)}{x^2}\right] = 2 \Rightarrow \lim\limits_{x \to 0}\frac{p(x)}{x^2} = 1$ ...(i)

$\Rightarrow P(0) = 0 \Rightarrow e = 0$

On applying L'Hospital rule to eqn. (i), we get

$\lim\limits_{x \to 0}\frac{p'(x)}{2x} = 1 \Rightarrow p'(0) = 0 \Rightarrow d = 0$

Again on applying L' H rule, we get

$\lim\limits_{x \to 0}\frac{p''(x)}{2} = 1 \Rightarrow p''(0) = 2 \Rightarrow c = 1$

$\therefore p(x) - ax^4 + by^3 + x^2$

$\Rightarrow p'(x) = 4ax^3 + 3bx^2 + 2x$

As p(x) has extremum at x = 1 and 2

$\therefore p'(1) = 0$ and $p'(2) = 0$

$\Rightarrow$ 4a + 3b + 2 = 0 ...(i)

and 8a + 3b + 1 = 0 ...(ii)

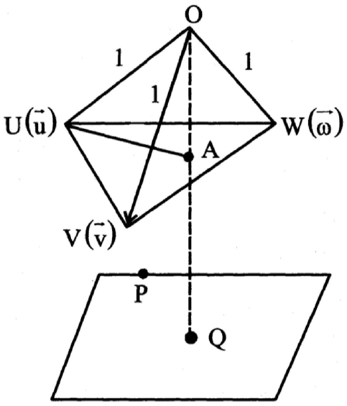
On solving (i) and (ii), we get $a = \frac{1}{4}$ and b = -1

$\therefore$ p(x) = $\frac{1}{4}x^4 - x^3 + x^2$

$\therefore$ p(2) = 0

47. 45.0

Explanation:



Given, $|\overrightarrow{\mathbf{u}} - \overrightarrow{\mathrm{v}}| = |\overrightarrow{\mathbf{v}} - \overrightarrow{\mathrm{w}} = |\overrightarrow{\mathbf{w}} - \overrightarrow{\mathrm{u}}|$

So, $\triangle$UVW is one equilateral triangle

Given that distances of points U, V, W from plane P $= \frac{7}{2} \Rightarrow \mathrm{AQ} = \frac{7}{2}$

Distance of plane P from origin

$= \left|\frac{0+0+0-16}{\sqrt{3+4+9}}\right| = 4 = \mathrm{OQ}$

$\therefore$ OA = OQ - AQ $= 4 - \frac{7}{2} = \frac{1}{2}$

In $\triangle$OAU, UA $=\sqrt{\mathrm{OV}^2 - \mathrm{OA}^2} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2} = \mathrm{R}$

In $\triangle$UVW, is circumcenter

US = R cos30° $\Rightarrow$ UV = 2 R cos 30° $= \frac{3}{2}$

$\therefore$ Ar $\triangle$UVW $= \frac{\sqrt{3}}{4}\left(\frac{3}{2}\right)^2 = \frac{9\sqrt{3}}{16}$

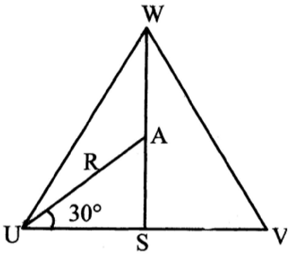
Volume of tetrahedron with coterminous edges

$\vec{u}, \vec{v}, \vec{w} = \frac{1}{3}$ (Ar $\triangle$UVW) $\times$ OA

$= \frac{1}{3} \times \frac{9\sqrt{3}}{16} \times \frac{1}{2} = \frac{3\sqrt{3}}{32}$

$\therefore$ Volume of parallelopiped:

$\mathbf{V} = 6 \times$ volume of tetrahedron $= \frac{6\times3\sqrt{3}}{32} = \frac{9\sqrt{3}}{16}$



Now, $\frac{80}{\sqrt{3}}\mathbf{V} = \frac{80}{\sqrt{3}} \times \frac{9\sqrt{3}}{16} = 45$

48. 5

Explanation:

From the given information, it is clear that

$\vec{a} = \frac{\hat{\mathbf{i}} - 2\hat{\mathbf{j}}}{\sqrt{5}}$

$\Rightarrow |\vec{a}| = 1, |\vec{b}| = 1, \vec{a} \cdot \vec{b} = 0$

Now, $(2\vec{a} + \vec{b}) \cdot [(\vec{a} \times \vec{b}) \times (\vec{a} - 2\vec{b})]$

$= (2\vec{\mathbf{a}} + \vec{\mathbf{b}}) \cdot \left[a^2\vec{\mathbf{b}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}) \cdot \vec{\mathbf{a}} + 2b^2 \cdot \vec{a} - 2(\vec{b} \cdot \vec{a}) \cdot b\right]$

$= [2\vec{\mathrm{a}} + \vec{\mathrm{b}}] \cdot [\vec{\mathrm{b}} + 2\vec{\mathrm{a}}] = 4\vec{\mathrm{a}}^2 + \vec{\mathrm{b}}^2$

= 4(1) + 1 = 5 [as $a \cdot b = 0$]